

Attendance Questions: Find the volume of each figure. If necessary, round your answer to the nearest tenth.

1. A square prism with base area 189 ft^2 and height 21 ft.

2. A regular hexagonal prism with base edge length 24 m and height 10 m

3. A cylinder with diameter 16 in. and height 22 in.

- I can learn and apply the formula for the volume of a pyramid.
- I can learn and apply the formula for the volume of a cone.

Common Core

CC.9-12.G.GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

CC.9-12.G.GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.

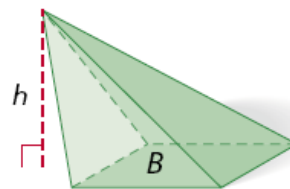
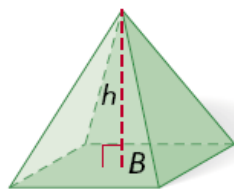
CC.9-12.G.MG.1 Use geometric shapes, their measures, and their properties to describe objects.

CC.9-12.G.MG.2 Apply concepts of density based on area and volume in modeling situations.

Volume of a Pyramid

The volume of a pyramid with base area B and height h

$$\text{is } V = \frac{1}{3}Bh.$$



Question: What has a circular base and is surrounded by water?

Answer: Coney Island

"Change is not made without inconvenience, even from worse to better."—
Theologian, Richard Hooker

1 Finding Volumes of Pyramids

Find the volume of each pyramid.

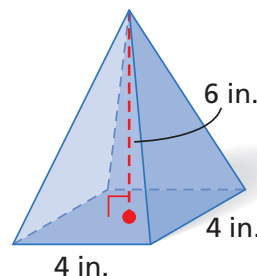
- A** a rectangular pyramid with length 7 ft, width 9 ft, and height 12 ft

$$V = \frac{1}{3}Bh = \frac{1}{3}(7 \cdot 9)(12) = 252 \text{ ft}^3$$

- B** the square pyramid

The base is a square with a side length of 4 in., and the height is 6 in.

$$V = \frac{1}{3}Bh = \frac{1}{3}(4^2)(6) = 32 \text{ in}^3$$



Find the volume of the pyramid.

- C** the trapezoidal pyramid with base $ABCD$, where $\overline{AB} \parallel \overline{CD}$ and $\overline{AE} \perp \text{plane } ABC$

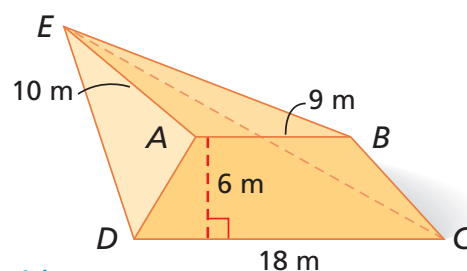
Step 1 Find the area of the base.

$$\begin{aligned} B &= \frac{1}{2}(b_1 + b_2)h && \text{Area of a trapezoid} \\ &= \frac{1}{2}(9 + 18)6 && \text{Substitute 9 for } b_1, 18 \text{ for } b_2, \text{ and 6 for } h. \\ &= 81 \text{ m}^2 && \text{Simplify.} \end{aligned}$$

Step 2 Use the base area and the height to find the volume.

Because $\overline{AE} \perp \text{plane } ABC$, \overline{AE} is the altitude, so the height is equal to AE .

$$\begin{aligned} V &= \frac{1}{3}Bh && \text{Volume of a pyramid} \\ &= \frac{1}{3}(81)(10) && \text{Substitute 81 for } B \text{ and 10 for } h. \\ &= 270 \text{ m}^3 \end{aligned}$$



Example 1. Find the volume of each pyramid.

A. A rectangular pyramid with length 11 m, width 18 m, and height 23 m.

B. A square pyramid with base edge length 9 cm and height 14 cm.

C. A regular hexagonal pyramid with height equal to the apothem of the base.

4. Guided Practice: Find the volume of a regular hexagonal pyramid with a base edge length of 2 cm and a height equal to the area of the base.

2 **Architecture Application**

The Rainforest Pyramid in Galveston, Texas, is a square pyramid with a base area of about 1 acre and a height of 10 stories. Estimate the volume in cubic yards and in cubic feet. (Hint: 1 acre = 4840 yd², 1 story ≈ 10 ft)

The base is a square with an area of about 4840 yd². The base edge length is $\sqrt{4840} \approx 70$ yd. The height is about $10(10) = 100$ ft, or about 33 yd.

First find the volume in cubic yards.

$$\begin{aligned} V &= \frac{1}{3}Bh && \text{Volume of a regular pyramid} \\ &= \frac{1}{3}(70^2)(33) = 53,900 \text{ yd}^3 && \text{Substitute } 70^2 \text{ for } B \text{ and } 33 \text{ for } h. \end{aligned}$$

Then convert your answer to find the volume in cubic feet.

The volume of one cubic yard is $(3 \text{ ft})(3 \text{ ft})(3 \text{ ft}) = 27 \text{ ft}^3$.

Use the conversion factor $\frac{27 \text{ ft}^3}{1 \text{ yd}^3}$ to find the volume in cubic feet.

$$53,900 \text{ yd}^3 \cdot \frac{27 \text{ ft}^3}{1 \text{ yd}^3} \approx 1,455,300 \text{ ft}^3$$

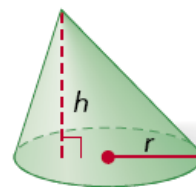
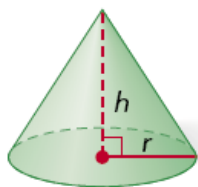


Example 2. An art gallery is a 6-story square pyramid with base area $\frac{1}{2}$ acre (1 acre = 4840 yd², 1 story \approx 10 ft). Estimate the volume in cubic yards and cubic feet.

5. Guided Practice: What would be the volume of the Pyramid Arena if the height were doubled?

Volume of Cones

The volume of a cone with base area B , radius r , and height h is $V = \frac{1}{3}Bh$,
or $V = \frac{1}{3}\pi r^2 h$.



3 Finding Volumes of Cones

Find the volume of each cone. Give your answers both in terms of π and rounded to the nearest tenth.

A a cone with radius 5 cm and height 12 cm

$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h && \text{Volume of a cone} \\ &= \frac{1}{3}\pi(5)^2(12) && \text{Substitute 5 for } r \text{ and 12 for } h. \\ &= 100\pi \text{ cm}^3 \approx 314.2 \text{ cm}^3 && \text{Simplify.} \end{aligned}$$

B a cone with a base circumference of 21π cm and a height 3 cm less than twice the radius

Step 1 Use the circumference to find the radius.

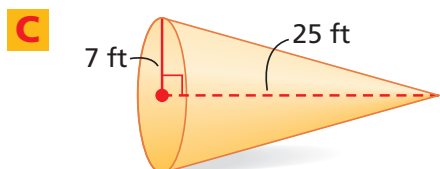
$$\begin{aligned} 2\pi r &= 21\pi && \text{Substitute } 21\pi \text{ for } C. \\ r &= 10.5 \text{ cm} && \text{Divide both sides by } 2\pi. \end{aligned}$$

Step 2 Use the radius to find the height.

$$2(10.5) - 3 = 18 \text{ cm} \quad \text{The height is 3 cm less than twice the radius.}$$

Step 3 Use the radius and height to find the volume.

$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h && \text{Volume of a cone} \\ &= \frac{1}{3}\pi(10.5)^2(18) && \text{Substitute 10.5 for } r \text{ and 18 for } h. \\ &= 661.5\pi \text{ cm}^3 \approx 2078.2 \text{ cm}^3 && \text{Simplify.} \end{aligned}$$



Step 1 Use the Pythagorean Theorem to find the height.

$$\begin{aligned} 7^2 + h^2 &= 25^2 && \text{Pythagorean Theorem} \\ h^2 &= 576 && \text{Subtract } 7^2 \text{ from both sides.} \\ h &= 24 && \text{Take the square root of both sides.} \end{aligned}$$

Step 2 Use the radius and height to find the volume.

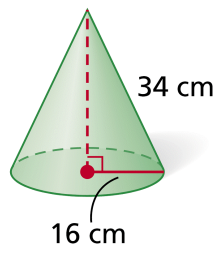
$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h && \text{Volume of a cone} \\ &= \frac{1}{3}\pi(7)^2(24) && \text{Substitute 7 for } r \text{ and 24 for } h. \\ &= 392\pi \text{ ft}^3 \approx 1231.5 \text{ ft}^3 && \text{Simplify.} \end{aligned}$$

Example 3. Find the volume of each cone. Give your answers both in terms of π and rounded to the nearest tenth.

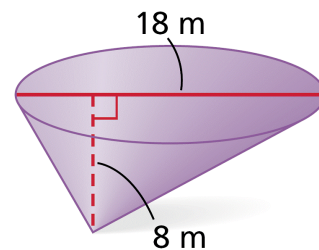
A. A cone with radius 7 cm and height 15 cm.

B. A cone with base circumference 25π in. and a height 2 in. more than twice the radius.

C.



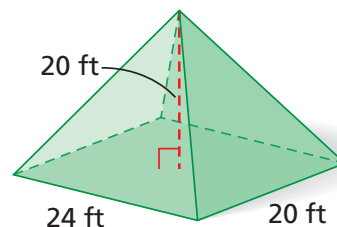
6. **Guided Practice:** Find the volume of the cone.



4

Exploring Effects of Changing Dimensions

The length, width, and height of the rectangular pyramid are multiplied by $\frac{1}{4}$. Describe the effect on the volume.



original dimensions:

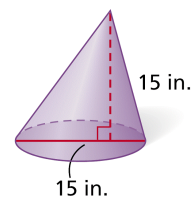
$$\begin{aligned} V &= \frac{1}{3}Bh \\ &= \frac{1}{3}(24 \cdot 20)(20) \\ &= 3200 \text{ ft}^3 \end{aligned}$$

length, width, and height multiplied by $\frac{1}{4}$:

$$\begin{aligned} V &= \frac{1}{3}Bh \\ &= \frac{1}{3}(6 \cdot 5)(5) \\ &= 50 \text{ ft}^3 \end{aligned}$$

Notice that $50 = \frac{1}{64}(3200)$. If the length, width, and height are multiplied by $\frac{1}{4}$, the volume is multiplied by $\left(\frac{1}{4}\right)^3$, or $\frac{1}{64}$.

Example 4. The diameter and height of the cone are divided by 3.
Describe the effect on the volume.



7. Guided Practice: The radius and height of the cone are doubled. Describe the effect on the volume.

5

Finding Volumes of Composite Three-Dimensional Figures

Find the volume of the composite figure. Round to the nearest tenth.

The volume of the cylinder is

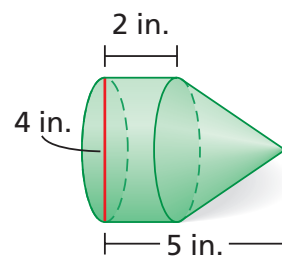
$$V = \pi r^2 h = \pi(2)^2(2) = 8\pi \text{ in}^3.$$

The volume of the cone is

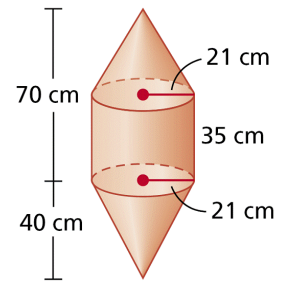
$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(2)^2(3) = 4\pi \text{ in}^3.$$

The volume of the composite figure is the sum of the volumes.

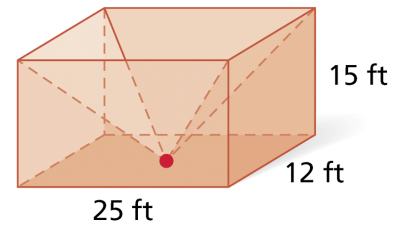
$$V = 8\pi + 4\pi = 12\pi \text{ in}^3 \approx 37.7 \text{ in}^3$$



Example 5. Find the volume of the composite figure. Round to the nearest tenth.



8. Guided Practice: Find the volume of the composite figure. Round the final answer to the nearest tenth.



11-3 Volumes of Pyramids and Cones (pp 762-764) 13, 15, 16, 17, 19-23, 34, 39-41.

Question: What has a circular base and is surrounded by water?

Answer: Coney Island



"Do you know why the volume of a cone is one-third the volume of the cylinder with the same height and base?"

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